An action of the gluodynamics string from the Wilson loop expansion

D.V.ANTONOV *
Institute of Theoretical and Experimental Physics,
B.Cheremushkinskaya, 25, 117 218, Moscow, Russia
and
Institut für Elementarteilchenphysik, Humboldt-Universität,
Invalidenstrasse 110, D-10115, Berlin, FRG

D.EBERT †
Institut für Elementarteilchenphysik, Humboldt-Universität,
Invalidenstrasse 110, D-10115, Berlin, FRG

Abstract

The nonabelian Stokes theorem, representing a Wilson loop as an integral over all the orientations in colour space, and the cumulant expansion are used for derivation of the string effective action in $SU(2)$ gluodynamics. The obtained theory is the theory of the rigid string interacting with the rank two antisymmetric Kalb-Ramond fields. In this model there exists a phase where there are no problems of crumpling and wrong high temperature behaviour of the string tension, which are present in the free rigid string theory. The Langevin approach to stochastic quantization of the obtained theory is applied.

*E-mail: antonov@pha2.physik.hu-berlin.de, antonov@vxitep.itep.ru. Supported by the Russian Fundamental Research Foundation, Grant No. 96-02-19184 and by the INTAS, Grant No.94-2851.

†E-mail: debert@qft2.physik.hu-berlin.de

1. Introduction

There exist several arguments why the Nambu-Goto string is a bad candidate for the gluodynamics string. First it contradicts with the parton-like behaviour observed in deep inelastic scattering, since the observed scattering amplitudes have a power law fall-off, while the Nambu-Goto string gives rise to the exponential fall-off. Second an averaged Wilson loop $< W(C) >$ is invariant under orientation-preserving reparametrizations of the oriented contour C: $< W(C(s)) >=< W(C(\alpha(s)) >$ when $\alpha'(s) > 0$, while the Nambu-Goto action is not sensitive to the sign of the reparametrization and therefore cannot be a string action corresponding to the Wilson loop expansion. Another fact of fundamental importance is that the gluodynamics string is an asymptotical theory, which exists only at the distances larger than the correlation length of the vacuum T_g [1]. This property can be taken into account if one adds to the Nambu-Goto term the so-called rigidity term[2,3], and it was shown in[4] that the rigidity term actually appears in the expansion of the averaged Wilson loop in powers of $\frac{T_g}{r}$, where r is the size of the Wilson loop. Such an expansion is valid in the confining regime of the Wilson loop, when the ratio $\frac{T_g}{r}$ is small[5]. This expansion allows one to establish the criterion of confinement in terms of the two scalar functions, which parametrize the bilocal correlator of gluonic fields[6].

However there are some obstacles which destroy the status of the rigid string as the gluodynamics string. The most serious of them is that the operator product expansion in the rigid string theory contains a local operator of dimenion two[7], while the operator product expansion in gluodynamics cannot involve any operators of dimension less than four. Second as a higher-derivative theory the rigid string theory has no the lowest-energy state[8]. Third if the β-function has no zeros the string world sheet is creased (normals are short-ranged), which leads to the so-called problem of crumpling[2,9] (for a review see[10]). In this case there presents a tachyon (the state with a negative norm) in the string spectrum[2,9]. The last problem of the rigid string theory is that at high temperatures the value of the free energy per unit length (string tension) does not coincide with the behaviour derived from gluodynamics[11]. The problems of nonexistence of a lowest-energy state, presence of a tachyon in the spectrum, and the wrong high temperature behaviour of the string tension are absent in the new nonlocal theory of a string with the negative stiffness, which was proposed in[12]. However it remains unclear how this model can be derived from the gluodynamics partition function or from the Wilson loop expansion.

An alternative approach to the gluodynamics string was suggested in[13], where it was shown that a phase transition occured in the rigid string theory coupled to the Kalb-Ramond fields[14], so that the new phase was characterized by the long range order of normals and therefore absence of crumpling. Moreover in[15] it was demonstrated that this model gave a consistent solution for the free energy of gluodynamics at high temperatures and thus cured the corresponding problem of the free rigid string theory.

The aim of this letter is to show how the theory of a rigid string, interacting with the Kalb-Ramond fields, may be derived via the expansion of the Wilson loop written through the nonabelian Stokes theorem and the cumulant expansion[6,16,20]. There exist in literature two versions of the nonabelian Stokes theorem: first was suggested in[16,17], and its important property is the presence of the path ordering, while in the second version, suggested in[18], the path ordering was replaced by the integration over an auxiliary scalar field from the $SU(N_c)/[U(1)]^{(N_c-1)}$ coset space. For the purposes stated above it occures crucial to use the second version of the nonabelian Stokes theorem.

After that we shall apply the Langevin approach[19] to stochastic quantization of the obtained

model. Using the Fourier transformation we shall find the retarded Green functions of the differential operators standing in the Langevin equations and reduce these equations to the integral-differential ones, which can be solved perturbatively.

The main results of the letter are summarized in the Conclusion.

2. Rigid string coupled to the Kalb-Ramond fields from the Wilson loop expansion and the Langevin approach to its quantization

We shall start with the nonabelian Stokes theorem, suggested in[18], and consider the $SU(2)$ case. Then one can write the Wilson loop $W(C)=trPexp\left(i\oint\limits_C A_\mu^a t^a dx_\mu\right)$ in the following way[18]

$$W(C)=\int D\vec{n}\ exp\left[\frac{iJ}{2}\left(-\int\limits_S d\sigma_{\alpha\beta}F_{\alpha\beta}^i n^i+\int\limits_S d\sigma_{\alpha\beta}\varepsilon^{ijk}n^i(D_\alpha\vec{n})^j(D_\beta\vec{n})^k\right)\right], \tag{1}$$

where $\vec{n}$ is a unit 3-vector which characterizes the instant orientation in the colour space, $J=\frac12$, $1, \frac32, ...$ is the spin of the representation of the Wilson loop considered, $F_{\alpha\beta}^i=\partial_\alpha A_\beta^i-\partial_\beta A_\alpha^i+$ $+\varepsilon^{ijk}A_\alpha^j A_\beta^k$, $D_\alpha^{ij}=\delta^{ij}\partial_\alpha-\varepsilon^{ijk}A_\alpha^k$, and S is any surface bounded by the contour C. It should be emphasized that the representation of the Wilson loop is fixed to be exactly J by the second term on the right hand side of equation (1) (the so-called Wess-Zumino term), which breaks down the isotropy of the $\vec{n}$-space. Therefore while dealing with the Wilson loop in a given representation, considering the integration over $\vec{n}$ in (1) as some averaging procerure and using the cumulant expansion[6,16,20] one gets in the bilocal approximation[6]

$$W(C)=exp\left[-\frac{iJ}{2}\int\limits_S d\sigma_{\alpha\beta}<G_{\alpha\beta}>_{\vec{n}}-\frac{J^2}{4}\int\limits_S d\sigma_{\alpha\beta}(x)d\sigma_{\mu\nu}(x')\ll G_{\alpha\beta}(x)G_{\mu\nu}(x')\gg_{\vec{n}}\right], \tag{2}$$

where $G_{\alpha\beta}=F_{\alpha\beta}^i n^i-\varepsilon^{ijk}n^i(D_\alpha\vec{n})^j(D_\beta\vec{n})^k$, so that the first term on the right hand side of equation (2) does not vanish (since the averaging is performed only over the $\vec{n}$-field, but not over the physical vacuum), and $<G_{\alpha\beta}>_{\vec{n}}$ is some antisymmetric tensor field.

In what follows we shall use the method suggested in[4] and consider $-lnW(C)$ with $W(C)$ defined via (2) as a gluodynamics string effective action. At this point we shall quote its resulting form (5), which is the action of the rigid string interacting with the Kalb-Ramond fields. Gaussian integration over the latter in (5) yields the long range Coulomb potential[13]

$$V(x(\xi)-x(\xi'),a)=\frac{2g_0}{\pi}\frac{1}{(x(\xi)-x(\xi'))^2+a^2\sqrt{g}},$$

with the coupling constant $g_0=\alpha_0\alpha_{Coulomb}\equiv\alpha_0\frac{e^2}{4\pi}$, where in order to avoid the singularity at $\xi=\xi'$ we introduced a cut-off a, which was reasonable to be taken of the order of the correlation length of the vacuum T_g. Let us then introduce a dimensionless field $B_\mu^i=aA_\mu^i$ and an auxiliary Abelian field H_μ, which satisfies the equation

$$\gamma\int d^4x\left(\varepsilon_{\mu\nu\lambda\rho}\mathcal{F}_{\mu\nu}\partial_\lambda H_\rho-\gamma H_\mu^2\right)=iJ\int\limits_S d\sigma_{\alpha\beta}\varepsilon^{ijk}<n^i(D_\alpha\vec{n})^j(D_\beta\vec{n})^k>_{\vec{n}}, \tag{3}$$

where γ is an arbitrary parameter, $\mathcal{F}_{\mu\nu}=<f_{\mu\nu}^i n^i>_{\vec{n}}$, $f_{\mu\nu}^i=\partial_\mu B_\nu^i-\partial_\nu B_\mu^i+\frac1a\varepsilon^{ijk}B_\mu^j B_\nu^k$. The dependence on the parameter γ drops out when one integrates over the field H_μ, which leads to the Lagrangian of the antisymmetric tensor field

$$\mathcal{L} = -\frac{1}{12} K_{\mu\nu\lambda} K_{\mu\nu\lambda}, \tag{4}$$

where $K_{\mu\nu\lambda} = \partial_\mu \mathcal{F}_{\nu\lambda} + \partial_\nu \mathcal{F}_{\lambda\mu} + \partial_\lambda \mathcal{F}_{\mu\nu}$. Parametrizing the bilocal cumulant $\ll G_{\alpha\beta}(x) G_{\mu\nu}(x') \gg_{\vec{n}}$ in the following way[6,16]

$$\ll G_{\alpha\beta}(x) G_{\mu\nu}(x') \gg_{\vec{n}} = (\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}) D\left(\frac{(x-x')^2}{T_g^2}\right) + \frac{1}{2}\left[\frac{\partial}{\partial x_\alpha}((x-x')_\mu \delta_{\beta\nu} - (x-x')_\nu \delta_{\beta\mu}) + \right.$$

$$\left. + \frac{\partial}{\partial x_\beta}((x-x')_\nu \delta_{\alpha\mu} - (x-x')_\mu \delta_{\alpha\nu})\right] D_1\left(\frac{(x-x')^2}{T_g^2}\right),$$

where D and D_1 are two coefficient functions, expanding the second term on the right hand side of (2) in powers of $\frac{T_g}{r}$, where r is the size of the Wilson loop, in the same manner as it was done in[4] and using (3) and (4), we finally get from equation (2) the following action of the $SU(2)$ gluodynamics string in the bilocal approximation

$$S_{biloc.} = \sigma \int d^2\xi \sqrt{g} + \frac{1}{2\alpha_0} \int d^2\xi \left[\frac{1}{\sqrt{g}} (\partial^2 x)^2 + \lambda^{ab}((\partial_a x_\mu)(\partial_b x_\mu) - \sqrt{g}\delta_{ab})\right] +$$

$$+ e_0 \int d^2\xi \varepsilon^{ab} (\partial_a x_\mu)(\partial_b x_\nu) \phi_{\mu\nu} + \frac{1}{12} \int d^4 y P_{\mu\nu\lambda} P_{\mu\nu\lambda} + O\left[max\left(\frac{T_g^6 D(0)}{r^2}, \frac{T_g^6 D_1(0)}{r^2}\right)\right]. \tag{5}$$

In (5) $\sigma = J^2 T_g^2 \int d^2 z D(z^2)$, $\alpha_0 = \frac{16}{J^2 T_g^4 \int d^2 z z^2 (2D_1(z^2) - D(z^2))}$, $e_0 = \frac{J}{2a}$, λ^{ab} is the Lagrange multiplier, $\phi_{\mu\nu} = i\mathcal{F}_{\mu\nu}$, $P_{\mu\nu\lambda} = iK_{\mu\nu\lambda}$, $g = det \parallel g_{ab} \parallel$, and we have used the conformal gauge $g_{ab} = \sqrt{g}\delta_{ab}$. One can now see that all the dependence on the spin of the representation of the Wilson loop has gone into the coupling constants.

Thus expanding the Wilson loop, written through the nonabelian Stokes theorem suggested in[18], we obtained an action of the $SU(2)$ gluodynamics string, which occured to be the action of the rigid string coupled to the rank two antisymmetric Kalb-Ramond field $\phi_{\mu\nu}$[14]. It was shown in[13] that there existed two phases in this theory, which were distinguished by the values of the order parameter m, defined from the mass gap equation

$$\int d^2 p \frac{p^2}{p^2(p^2+m^2) + p^2 V_0(p) + V_1(p)} = \frac{2\pi^2}{\alpha_0},$$

where $V_0(p) = 8g_0 K_0(a|p|)$, $V_1(p) = \frac{8g_0}{a^2}(a|p|K_1(a|p|) - 1)$, K_0 and K_1 were the Macdonald functions. In the disordered phase, corresponding to the values $m > 0$, the coupling constants α_0 and g_0 are fixed by dimensional transmutation in terms of m and the cut-off a, while the phase corresponding to $m = 0$ is characterized by the long range order of normals (absence of crumpling) and therefore may describe the gluodynamics string, as it could be expected. Notice also that in[21] the renormalized mass gap equation was derived, and it was shown that the phase transition survived quantum fluctuations.

To conclude with we shall apply the Langevin approach[19] to stochastic quantization of our theory. To this end, by making use of the conformal gauge, we shall rewrite the Nambu-Goto term on the right hand side of equation (5) in an equivalent form[22] $\frac{\sigma}{2} \int d^2\xi (\partial^a x_\mu)(\partial_a x_\mu)$ and then

neglect for simplicity the rigidity term, since it is of the highest order in T_g in comparison with the others. Therefore one gets from (5) the following Langevin equations

$$\dot{x}_\mu - \sigma\partial^a\partial_a x_\mu = \eta_\mu - \frac{e_0}{3}P_{\mu\nu\lambda}\varepsilon^{ab}(\partial_a x_\nu)(\partial_b x_\lambda), \tag{6}$$

$$\dot{\phi}_{\mu\nu} - (\partial_\alpha\partial_\alpha\phi_{\mu\nu} + \partial_\mu\partial_\lambda\phi_{\nu\lambda} + \partial_\nu\partial_\lambda\phi_{\lambda\mu}) = \eta_{\mu\nu} - e_0\delta^{(2)}(\xi_\perp)\varepsilon^{ab}(\partial_a x_\mu)(\partial_b x_\nu), \tag{7}$$

where in (6) the Langevin time t has the dimension of the fourth power of length while in (7) it has the dimension of the square of length, η_μ and $\eta_{\mu\nu}$ are two Gaussian noises, whose bilocal correlation functions have the form $< \eta_\mu(x,t)\eta_\nu(x',t') >= 2\delta_{\mu\nu}\delta(x-x')\delta(t-t'), < \eta_{\mu\nu}(x,t)\eta_{\alpha\beta}(x',t') >=$ $= 2(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})\delta(x-x')\delta(t-t')$, and $\xi^a_\perp$ is a hyperplane perpendicular to the hyperplane ξ^a.

The retarded Green functions of the operators standing on the left hand sides of equations (6) and (7) can be obtained via the Fourier transformation, and one gets from (6) and (7) the following integral-differential equations

$$x_\mu(\xi,t) = \int\limits_0^t dt' \int d^2\xi' G(\xi-\xi', t-t')\Bigg[\eta_\mu(\xi',t') - \frac{e_0}{3}P_{\mu\nu\lambda}(x(\xi',t'),t')\varepsilon^{ab}\left(\frac{\partial}{\partial\xi'^a}x_\nu(\xi',t')\right)\cdot \left(\frac{\partial}{\partial\xi'^b}x_\lambda(\xi',t')\right)\Bigg], \tag{8}$$

$$\phi_{\mu\nu}(x,t) = \int\limits_0^t dt' \int d^4x' \mathcal{G}_{\mu\nu,\alpha\beta}(x-x', t-t')\Big[\eta_{\mu\nu}(x',t') - e_0\delta^{(2)}(\xi_\perp)\varepsilon^{ab}(\partial_a x'_\mu)(\partial_b x'_\nu)\Big], \tag{9}$$

where the Green functions $G(\xi,t)$ and $\mathcal{G}_{\mu\nu,\alpha\beta}(x,t)$ read

$$G(\xi,t) = \frac{e^{-\frac{\xi^2}{4\sigma t}}}{4\pi\sigma t},$$

$$\mathcal{G}_{\mu\nu,\alpha\beta}(x,t) = \frac{1}{96}\Bigg[(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})\left(\frac{e^{-\frac{x^2}{4t}}}{\pi^2 t^2} + 32\delta(x)\right) + \frac{1}{x^2}\Big(\delta_{\mu\alpha}x_\nu x_\beta + \delta_{\nu\beta}x_\mu x_\alpha - \delta_{\mu\beta}x_\nu x_\alpha - \delta_{\nu\alpha}x_\mu x_\beta\Big)\cdot \left(\frac{e^{-\frac{x^2}{4t}}}{\pi^2 t^2} - 16\delta(x)\right)\Bigg],$$

and the initial conditions $x_\mu(\xi,0) = 0, \phi_{\mu\nu}(x,0) = 0$ were implied. Solving equations (8) and (9) perturbatively one can develop stochastic diagrammatic technique in the model under consideration in the coordinate representation.

3. Conclusion

In this letter we have shown how the Wilson loop expansion in $SU(2)$ gluodynamics generates string effective action. To this end we have written the Wilson loop via the nonabelian Stokes theorem suggested in[18] and applied to it the cumulant expansion[6,16,20] in the bilocal

approximation[6]. After that, introducing an auxiliary Abelian field, which satisfies integral-differential equation (3), and integrating it over, we eliminated the coupling of the rigidity string term with the Wess-Zumino term, which fixed the representation of the Wilson loop. Then, Taylor expanding the second term of the cumulant expansion in powers of $\frac{T_g}{r}$, where T_g is the correlation length of the vacuum[5,6], and r is the size of the Wilson loop, we arrived at the action of the rigid string interacting with the rank two antisymmetric Kalb-Ramond tensor fields. This interaction is the long ranged Coulomb interaction, which should be cut off at the distances of the order of T_g, and its coupling constant is proportional to the spin of the representation of the Wilson loop and inversly proportional to the Coulomb cut-off. The dependence on the spin of the representation also appears in the Nambu-Goto string tension and the coupling constant of the rigidity term in contrast to[4], where another version of the nonabelian Stokes theorem[16,17] was applied to derivation of the string effective action.

The obtained theory provides correspondence between its coupling constants and the gluodynamics coupling constant at high temperatures[15] in contrast to the free rigid string theory[11] and possesses a phase, where the normals to the string world sheet are long ranged[13] and therefore is a much better candidate for the gluodynamics string than the free rigid string.

Finally we derived the Langevin equations in this theory, where the highest in T_g rigidity term was neglected. After that we found the retarded Green functions of the differential operators, standing in these equations, and reduced the problem to the system of two integral-differential equations, which can be solved perturbatively.

4. Acknowledgements

D.A. is deeply grateful to Professors Yu.M.Makeenko and Yu.A.Simonov for useful discussions and to the theory group of the Quantum Field Theory Department of the Institut für Physik of the Humboldt-Universität of Berlin for kind hospitality.

References

1. Yu.A.Simonov, ITEP-28-92, *Nuovo Cim.* **A107**, 2629 (1994).
2. A.M.Polyakov, *Nucl.Phys.* **B268**, 406 (1986).
3. H.Kleinert, *Phys.Lett.* **B174**, 335 (1986); T.L.Curtright et al., *Phys.Rev.* **D34**, 3811 (1986); G.Germán, *Mod.Phys.Lett.* **A6**, 1815 (1991).
4. D.V.Antonov et al., *Mod.Phys.Lett.* **A11**, 1905 (1996) (DESY-96-134).
5. M.Campostrini et al., *Z.Phys.* **C25**, 173 (1984); I.J.Ford et al., *Phys.Lett.* **B208**, 286 (1988); A. Di Giacomo and H. Panagopoulos, *Phys.Lett.* **B285**, 133 (1992); E.Laermann et al., *Nucl.Phys.* **B26** (Proc.Suppl.), 268 (1992).
6. H.G.Dosch, *Phys.Lett.* **B190**, 177 (1987); Yu.A.Simonov, *Nucl.Phys.* **B307**, 512 (1988); H.G.Dosch and Yu.A.Simonov, *Phys.Lett.* **B205**, 339 (1988), *Z.Phys.* **C45**, 147 (1989); Yu.A.Simonov, *Nucl.Phys.* **B324**, 67 (1989), *Phys.Lett.* **B226**, 151 (1989), *Phys.Lett.* **B228**, 413 (1989); for a review see Yu.A.Simonov, *Yad.Fiz.* **54**, 192 (1991).
7. A.M.Polyakov, unpublished.
8. S.W.Hawking, in *Quantum Field Theory and Quantum Statistics*, eds. I.A.Batalin et al. (Hilger, 1987); E.Braaten and C.K.Zachos, *Phys.Rev.* **D35**, 1512 (1987).
9. A.M.Polyakov, *Gauge Fields and Strings* (Harwood, 1987).
10. F.David, *Phys.Rep.* **184**, 221 (1989).
11. J.Polchinski and Z.Yang, *Phys.Rev.* **D46**, 3667 (1992).
12. H.Kleinert and A.M.Chervyakov, *hep-th* /9601030.
13. M.Awada and D.Zoller, *Phys.Lett.* **B325**, 115 (1994).
14. M.Kalb and P.Ramond, *Phys.Rev.* **D9**, 2273 (1974).
15. M.Awada, *Phys.Lett.* **B367**, 270 (1996).
16. Yu.A.Simonov, *Yad.Fiz.* **50**, 213 (1989).
17. M.B.Halpern, *Phys.Rev.* **D19**, 517 (1979); I.Ya.Aref'eva, *Theor.Math.Phys.* **43**, 111 (1980).
18. D.Diakonov and V.Petrov, in *Nonperturbative approaches to QCD*, Proceedings of the International Workshop at ECT*, Trento, July 10-29, 1995, ed. D.Diakonov (PNPI, 1995).
19. G.Parisi and Y.-S.Wu, *Sci.Sin.* **24**, 483 (1981).
20. N.G.Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland Physics Publishing, 1984).
21. M.Awada, *Phys.Lett.* **B351**, 468 (1995).
22. M.Green, J.Schwarz, E.Witten, *Superstring Theory* (Cambridge University Press, 1987).